Investigation of Ultrawideband Pulses in Wideband Helix Traveling Wave Tubes: Theory and Simulation*

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Research Goals and Approach

• Investigate the basic phenomena associated with multi-frequency, transient operation of the TWT. Some of which include,
  – linear dispersionless gain with frequency
  – frequency dependent phase velocity, coupling impedance, and attenuation
  – reflections
  – nonlinear effects
  – transient effects

• Two models will be used in this study*. They are a
  – 1D Linear Quasi-Analytic TWT Model
  – 1D Non-Linear Numerical TWT Model

* Models will be Validated against the experimental tube XWING. See the separate poster for details on the tube.
1D, Linear, Quasi-analytic TWT Model

The determinantal equation for analysis of the 1D linear TWT with space charge, from Gilmour is,

\[
\frac{\beta_e}{(\beta - \beta_e)^2} \left[ \frac{\beta^2 \beta_e}{(\beta^2 - \beta_e^2)} \right] 2C^3 - \frac{R^2 \beta_p^2}{\beta_e} + 1 = 0
\]

If given

\[ V_o, I_o, v_p(\omega), Z_o(\omega), R \]

of a TWT, you can calculate

\[
\beta_e = \frac{?}{v_p(\omega)} \quad \beta_e = \frac{?}{u_o} \quad \beta_p = \frac{?}{u_o} \quad C = \left( \frac{I_o Z_o(\omega)}{4V_o} \right)^{\frac{1}{3}}
\]

and solve for the roots of the determinantal equation.
4 Component Waves

This yields four harmonic eigenmode or “component“ waves that can be simultaneously excited when a pure-harmonic, steady state signal is applied to the input of the TWT.

In the gain regime 2 roots will be real and the others will be complex conjugates

\[ \beta = \beta_1, \beta_2, \beta_3, \beta_4 \]

Picture as 4 separate waves propagating on the structure.

- backward propagating wave
- forward propagating wave
- forward propagating wave which is exponentially growing
- forward propagating wave which is exponentially decaying

Note: Normal TWTs do no stimulate the backward wave due to the way the wave is introduced onto the helix. For these twts backward waves are only of concern when there are reflections or they are included in the models. This model does not deal with reflections so only the 3 forward waves will be used in the analysis.
Propagation Constants

With these propagation constants the electric field as a function of position and frequency can be determined from the equation

$$E(z, \omega) = \sum_{i=1}^{3} E_i F(\omega) e^{j(\omega t - \beta_i z)}$$

where $F(\omega)$ is the Fourier transform of the total time-dependent electric field signal, input into the TWT (the input signal) and $E_i$ is the weighting factor which determines how much of the total signal amplitude each component wave has. $E_i$ must be determined from the partitioning of energy consistent with relevant boundary conditions.
Equation Determinantal

In actual TWTs there are variations in the physical parameters of the TWT to maximize gain or efficiency. Adding z dependence to the determinantal equation to account for changes in the physical parameters yields,

\[
\frac{\beta_e(?)}{(\beta - \beta_e(?))^2} \left[ \frac{\beta^2 \beta_e(? , z)}{\beta^2 - \beta_e(? , z)^2} \right] 2C(? , z)^3 - \frac{R(?)^2 \beta_p^2}{\beta_e(?)} + 1 = 0
\]

where \( \beta_e \), and \( C \) are now,

\[
C(? , z) = \left[ \frac{I_o Z_o (?, z)}{4V_o} \right]^{\frac{1}{3}} \quad \beta_e(?, z) = \frac{?}{v_p(?, z)}
\]
Electric Field in Time Domain

Since this new determinantal equation is dependent on $z$, the propagation constants are also $z$-dependant and this must be accounted for in $E(z, \omega)$,

$$E(z, \omega) = \sum_{i=1}^{3} E_i F(\omega) e^{j \left( \frac{\omega t}{\beta_i} - \int_{0}^{z} \beta_i dz \right)}$$

The signal at every point in space and time can be determined from integrating over the frequency domain.

$$E(z, t) = \int_{-\infty}^{\infty} \sum_{i=1}^{3} E_i F(\omega) e^{j \left( \frac{\omega t}{\beta_i} - \int_{0}^{z} \beta_i dz \right)} d\omega$$
Numerical Approximations and Solution in Mathematica

• Low frequencies propagate without interaction with beam.
  The model is not valid when the wavelength of the signal is larger than the interaction
  regime of the TWT. These frequencies will be propagated as if no beam were present.

• Mathematica does not handle the integration well, so cast as ODE.
  Integrate over $\omega$, by solving

$$\frac{dH(z,\omega)}{d\omega} = \sum_{i=1}^{3} E_i F(i) e^{\left(i \omega t - \int_{0}^{z} \beta_i dz \right)} = E(z,\omega)$$

where we solve for $H(z,\omega)$ with the initial condition $H(z,0) = 0$. Then
$E(z,t) = \text{Re}(2^*H(z,\omega=\omega_i))$.

• Calculation time is too long due to the solution of $b$ in the integral.
  Create a table of $\beta$ in $\omega$ and $z$. Use an interpolating function to determine $\beta$ used
  in integration.
• **Incorporate Attenuation**  
  Originally attenuation was to be introduced into the determinantal equation by replacing $\beta_c$ with

$$\beta'_c(?,z) = \frac{?}{v_p(?,z)} - ja(?,z)$$

However, this is inconsistent with original derivation of determinantal equation, which assumed a lossless circuit to represent the fields, so re-derive new determinantal equation using a lossy circuit to represent the fields.

• **Calculate initial amplitudes of the three waves, $E_i$.**
  - From boundary conditions
  - CHRISTINE code plus basic theory
1D Non-linear Numerical TWT Model

The telegrapher’s equations are a pair of coupled differential equations that describe the rf current and voltage on a transmission line using distributed capacitance and inductance.

\[
\frac{\partial V}{\partial t} = -\frac{1}{C} \frac{\partial I}{\partial z} \quad \quad \quad \quad \frac{\partial I}{\partial t} = -\frac{1}{L} \frac{\partial V}{\partial z}
\]

The equations are derived from an analysis of the equivalent distributed circuit model, shown below, for a lossless, dispersionless, transmission line.
Velocity and Impedance

The phase velocity and characteristic impedance of the wave traveling on this line are

\[ v_p = \frac{1}{\sqrt{LC}} \quad K = \sqrt{\frac{L}{C}} \]

As is apparent from these equations, this model is dispersionless. All frequencies propagate with the same velocity.
Addition of Dispersion

- Adding frequency-dependent dispersion to the time-domain Telegrapher’s equations (TE) is not feasible. Causality will lead to artificial loss.

- So add beta- (propagation constant) dependent dispersion to the time-domain TE. If the circuit has no inherent gain or loss this can be done.

- L and C become beta dependent and the spatial derivative of the voltage and the spatial derivative of the current are multiplied, in the spatial frequency domain, by $L(\beta)$ and $C(\beta)$, respectively.
**Discretized Versions of TEs**

Discretized versions of the TEs with dispersion are shown below

\[
V_i^t = V_i^{t-1} - \frac{t}{\Delta x} G[C^{-1}(\beta), (I_{i+1/2}^{t-1/2} - I_{i-1/2}^{t-1/2})]
\]

\[
I_{i-1/2}^{t+1/2} = I_{i-1/2}^{t-1/2} - \frac{t}{\Delta x} G[L^{-1}(\beta), (V_i^t - V_{i-1}^t)]
\]

Here \( G \) is the operator

\[
G[a, b] = F^{-1}[aF(b)]
\]

where \( F \) is the Fourier transform.
This graph compares a desired vp/c with a vp/c estimated from the results of a cold circuit simulation. As can be seen agreement is very good, except at 1 GHz. This is likely due to estimation error.
Particle Solution

Macro particles, which represent a number of electrons and which have the same charge to mass ratio as an electron, are launched from the gun and propagated along the axis using the leapfrog method to determine position and velocity,

\[ v^{n+1} = v^n + a^{n+\frac{1}{2}} \Delta t \]
\[ x^{n+\frac{1}{2}} = x^{n-\frac{1}{2}} + v^n \Delta t \]

The force on the beam electrons is calculated from the electric field determined from the sum of the gradient of the line voltage and the gradient of the space charge potential.

\[ E = -\left( \frac{\partial V}{\partial z} + R \frac{\partial f}{\partial z} \right) \]
Work To Be Completed

- **Boundary Conditions**
  Perfectly Matched Layer (PML) boundary condition doesn’t seem to work with this type of dispersion formulation. Need to modify PML or determine new boundary scheme.

- **Map experimental input signal to the simulated voltage at the input of the line**

\[
Z_0 \propto \frac{\langle E_z \rangle^2}{P} \quad \quad \langle E_z \rangle = \frac{\partial V}{\partial z}
\]

Since power is conserved, if we know the reflection off the input coupler and know the power in, then we know how much power goes into the waveguide and from the interaction impedance and the definition of the line voltage, we can calculate the simulated voltage at input of line.
Results and Current Status

Compare Christine, Quasi-Analytic model, and Numerical Simulation

Determine accuracy of models by comparison with CHRISTINE. CHRISTINE 1D is a one-dimensional, non-linear, multi-frequency model of a TWT. It uses macro-particles and integrates wave and particle equations using a Lagrangian formulation. It was developed by the Naval Research Lab (NRL) several years ago and has been favorably compared to experiment.

Because of undetermined initial electric fields for the three component waves in the 1D, Linear, Quasi-analytic TWT Model, and inability to simulate attenuation, we will use growth rate for the moment as measure of accuracy for the XWING tube and ignore circuit attenuation.

Growth rate:

$$\exp \left[ j (t - \beta_i z) \right] \exp^{\beta_iz}$$

The $\beta_1$ term is called the growth rate.
Compare Growth Rates

Comparison of growth rate vs. frequency for Christine 1D, the quasi-analytic model and the non-linear time-domain simulation.

Growth rate of the quasi-analytic model and the numerical simulation match well with Christine 1D.

At higher frequencies, however, they are significantly different.
Compare Gain vs Frequency

Comparison of gain vs. frequency for Christine 1D and the non-linear time-domain simulation.

Total gain of the numerical nonlinear time-domain model matches well at lower frequencies with Christine 1D.

At higher frequencies, however, they are significantly different.

Discrepancy of gain and growth rate may be due to calculation of $R$ – plasma frequency reduction factor.
Summary of Current Status

1D Linear Quasi-analytic Model
Initial formulation has been coded into a program in Mathematica.
Issues to be dealt with
- Need to develop a method of incorporating Attenuation
- Determination of Initial Electric field for the three wave components

1D Nonlinear Numerical Model
Model is being developed and incorporates the initial implementation of most of the necessary phenomena needed to model the XWING tube.
Issues to be dealt with
- Modification of PML to reduce reflection
- Source with voltage as opposed to current
- Map experimental input signal to simulated line voltage